The Gains of Voronoi Partitioning in Asymmetric Warfare Logistics Resupply Management: A Case Study of Nigeria Military Onslaught Against Insurgence in North East Nigeria

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Abstract

This study develops a Voronoi-Based Heuristic Algorithm (VHA) to optimize the placement of Forward Operating Bases (FOBs) for improved logistics support in asymmetric warfare, focusing on the Nigerian Army's operations in Borno State. The primary objective is to enhance the efficiency, responsiveness and security of military supply chains by minimizing delivery time and distance to troops in dispersed and high-risk zones.

The research utilized data from military personnel, field surveys, and geographic mapping to model the operational environment using Voronoi diagrams. These diagrams partitioned the area into zones of influence for potential FOBs. The heuristic algorithm was designed to select optimal FOB locations based on factors such as proximity to demand points, engagement frequency, terrain, and ambush risk. Mathematical models were employed to estimate distance metrics and the cost of engagements.

The findings showed that the optimized VHA model reduced average resupply distances by 25% (from 30 km to 22.5 km) and decreased ambushes in Bama from 6 to 2 per 50 logistics trips which is a 66.7% improvement. The engagement casualty cost also dropped by 22.9%, from N412.2Cc to N317.9Cc. Furthermore, the number of required FOBs was reduced from 13 to 8, achieving a 38.5% decrease while maintaining effective operational coverage. The study concludes that the VHA provides a cost-effective and scientifically grounded method for strategic FOB placement in complex environments. It recommends routine updates to FOBs locations using real-time data and terrain analysis to support future military operations.

Keywords: *Voronoi: Asymmetric: Logistics: Convex Polygon: Euclidean Distance: Dirichlet Tessellation: Forward Operating Bases.*

1. INTRODUCTION

National security is the primary responsibility of any state, ensuring the protection of lives, territorial integrity, and national sovereignty. In Nigeria, this responsibility has been continually challenged by threats such as terrorism, banditry, and insurgency, particularly in states such as Borno in the North-East region (John, 2022). The military plays a central role in maintaining national security, especially in internal conflicts and counterinsurgency operations. Contemporary operations emphasize the critical importance of logistics, which, alongside strategy and tactics, forms the tripod of military success. General Dwight D. Eisenhower famously remarked that wars are often won or lost due to logistics (Enache & Voicila, 2020). Historical evidence reinforces this point. Military leaders like Napoleon and George Washington experienced significant setbacks due to logistical failures (Kress, 2002). NATO defines logistics as the science of planning and maintaining the movement of forces. General Pagonis further described logistics as the careful integration of transport, procurement, maintenance, and automation (Currie, 1995).

Since independence, the Armed Forces of Nigeria (AFN) have consistently undertaken Internal Security Operations (ISOs), including the civil war (1967–1970), Niger Delta militancy (1999–2007), and the Boko Haram insurgency (Premium Times, 2024). In response to the ongoing threat from Boko Haram and ISWAP, the Nigerian military launched Operation Lafiya Dole, now Operation Hadin Kai, focusing on degrading terrorist capabilities and stabilizing the region (Dawodu, 2023). Despite tactical gains, logistical challenges remain. Boko Haram operates asymmetrically, deploying IEDs, ambushes, and suicide bombers, often disrupting military supply chains and troop mobility. These challenges highlight a gap in the current logistics model, particularly the inability of Forward Operating Bases (FOBs) to respond quickly to frontline logistics needs due to poor placement and coordination.

To address these issues, the Nigerian Army introduced Logistics Bases (LBs) and FOBs, aimed at decentralizing supply distribution. However, these bases are not always optimally located, leading to delays and inefficiencies. A scientific approach using Voronoi Heuristic Algorithms (VHA) is proposed as a solution to optimize FOB locations. VHA partitions the operational area into regions where each unit or task team is closest to a specific FOB, reducing travel time and improving supply responsiveness.

1.1 Voronoi Diagrams and Their Applications in Facility Locations Problems.

Aggarwal et al., (1990) explained that Voronoi diagram involves the partitioning of a plane with n points into a convex polygon such that each polygon contains exactly one generating point and every point in a given' polygon is closer to its generating point than to any other. A Voronoi diagram is sometimes also known as a Dirichlet tessellation. The Cells are called Dirichlet regions, Thiessen polytopes or Voronoi polygons. Originally, it characterizes regions of proximity for a set of K sites in the plane where distance of points is defined by their Euclidean and optimal algorithms exist to compute the Voronoi diagram in and the Voronoi diagram can be represented in space (Berg et al.,2000). Voronoi diagrams have found application since the 17th century in various fields of human endeavor. Boicea (2010) submitted that Voronoi diagrams were considered as early at 1644 by René Descartes and were used by Dirichlet in 1850 in the investigation of positive quadratic forms. Voronoi studied the diagrams in 1907, and extended their application to higher dimensions. They are widely applied in areas such as computer graphics, epidemiology, geophysics and meteorology. One remarkable use of a Voronoi diagram in the past was by a physician John Snow who was able to identify the main source of cholera epidemic in London in 1854 by relating the deaths resulting from cholera to closeness to a particular infected water pump on Broad Street (Campbell, 2019).

The fundamental goal in any facility location issue is to wisely put a bunch of facilities, serving a bunch of users (or request points), to such an extent that specific optimality models are fulfilled. Facilities can be fixed or portable/mobile. The arrangement of users, then again, is discrete, comprising of limitedly numerous points, or constant, i.e., a locale where each point is viewed as a client. Given that all the facilities are similarly prepared altogether; a client consistently gets the service from his closest facility. In this manner, every facility has its service zone Z(a), comprising of the arrangement of users that are served by it.

The Voronoi graph of a limited arrangement of articles is a central mathematical design that partitions the implanting space into districts, every locale comprising of the points that are more like a given item than to the others. We may characterize numerous variations of Voronoi graphs relying upon the class of items, the distance capacities and the installing space. Nielsen et al (2007), explore a system for characterizing and building Voronoi outlines for an expansive class of distance capacities called Bregman divergences. Bregman divergences incorporate the conventional (squared) Euclidean distance yet in addition different difference estimates dependent on entropic capacities. Akkihal and Soni (2014) initial attempt to solve the problem of locating DCs used an Algorithm to Solve Uncapacitated Facility Location Problem (UFLP) by applying a demand function based on population needs. The model was based on discrete UFLP, such that each demand point is an aggregate of the demand in the area. Numerous researches on the aspects of locating medical facilities such as ambulance hospitals and other facilities in disaster had been undertaken in the past. Researches in location of logistics nodes in humanitarian operations and in business supply chain were relatively few and new. However, one of the approaches used in these location problems is from the geographical standpoint with a view to delivery of efficient and effective service. Models defining the problems are analyzed to determine the best locations for logistics nodes such as supply points, distribution centers, logistics hubs and warehouse. In operation research, location problems are mathematical models describing optimization problems here one or more facilities need to be placed in relation to a given set of customers or demand points (Kuhn and Klose, 2005). Single source capacitated facility problem (SSCFP) consists of opening a set of facilities and allocating the customers demand to those facilities in such a way that capacity of each facility is respected and each customer is serviced from one facility (Cachon and Terwiesch, 2009). In the case of discrete location problems, it is assumed that a finite set of potential facility sites is given and a decision is to be made as to which of this potential facility site is to be used (Charles and Lee., 2010).
Despite the numerous benefits of facility location models in logistics such in disaster management, emergency and humanitarian's operation, it's application in the military operations in asymmetric warfare setting has not been studied to a reasonable extent hence leaving several potential benefits untapped. Earliest known studies as regards application of Voronoi diagram in the military was for resources deployment (Zeimpeki et al, 2015). The link between the Voronoi diagrams and resource deployment comes from a simplification of the resource allocation problem (Tavares and Santos, 2018). It could be considered as allocating resources to some form of logistics hubs, train stations, military bases, power plants. However, no much studies have been done on locating

logistics nodes using the Voronoi diagram in asymmetric setting. The work is therefore focused on moving the frontiers of knowledge forward into the military arena by researching into applicability of using facility location models in locating FOBs to ensure effective and efficient logistics support to troops in asymmetric warfare using the ongoing CTCOIN in North East of Nigeria as benchmark.

As a concept, FOB refers to a permanently manned and well protected Main Operating Base (MOB) from where fighting troops are projected to carry out various tasks against insurgents and the logistics elements required in sustaining operations. The FOB get resupply of logistics materiel from logistics bases which are in turn supplied from bases depots, ports and airports. FOB locations are the last mile of the logistics supply chain in the OPHK. Currently, there are other logistics and tactical factors such as available infrastructure, amenities or combat advantage in terms of geography. However, no appropriate consideration is given to distances of the logistics last mile which is a major determinate of response time or resupply time for meeting troops logistics requirement. Consequently, the approach of this work is to either select facilities from a set of locations that have the desired tactical, technical and infrastructural characteristics or finding the strategic locations where distribution centers or facilities could be located and then identify which of these strategic locations possesses the desired tactical, technical and infrastructural characteristics of FOBs.

Geographically, we are looking for a possible location that could be represented by a vector Xi with coordinates (xil, xi2) in a 2 - dimensional Cartesian spaces. The main objective is to define the coverage area (Area of Responsibility) of a FOB such that the area satisfies certain criteria that are unique in the theatre of operation. According to (Balcik and Beamon,2008) model with coverage objectives are most suitable ones when response to time is the focal point or primary objective. In these models, coverage area of FOB is defined as the ability to reach demand point (logistics node) from a FOB (distribution center) within a specified response time. The specified response time is considered the fastest or optimal for resupplying any location with the Area of Responsibility (AOR) of a FOB when compared to the locations of other FOBs. According the condition for minimizing resupply time and cost is satisfied at the Voronoi region around the chosen facility (in this case FOB) and the model formulation would therefore be approach using a heuristic based on the Voronoi diagram (Yushimito et al, 2015).

2. METHODOLOGY

The Methodology adopted in this study is a combination of both quantitative and qualitative techniques. The level of research is descriptive, based on observing trends as well as examining and analyzing same. The Study adopted the field survey research design. structured and unstructured interviews as well as available statistics would be used to generate data. The efficient analysis of the data through the application of the appropriate Voronoi model with some modifications to suite the case understudy is expected to address the research.

2.1 Method of Data collection

Methods of Data Collection. Primary data would be collected through field survey by conducting semi- structured and unstructured face-to-face and telephone interviews as well as administering questionnaires in Abuja, Yobe, Maiduguri. Secondary data was also obtained through documents analysis from libraries, archives and the internet.

2.2 Model Development

2.2.1 Definition and Review of the Voronoi Diagram framework

The Voronoi diagram of a finite set of objects is a fundamental geometric structure that subdivides the embedding space into regions, each region consisting of the points that are closer to a given object than the other (Francis and White, 1974). It is an important mathematical and geometric construct with wide application in various fields due to its intuitive and efficient attributes for the allocation and partitioning of space (Xin and Murray, 2018). It is an essential facility allocation tools with respect to demand points. The objective of a facility allocation problem for which the Voronoi diagram may be deployed is to minimize the total distance from sited facilities to demand points with often the constraint that only one facility can be associated with each demand (Jean et al., 2007; Xin and Murray, 2018). The Voronoi diagram is structured in a manner that allows the generator (the facility servicing the demand points) to have the shortest distance to any demand point, if such a demand point is within the corresponding Voronoi polygon (Darusz, 2016). The optimal solution to an allocation problem is only achieved if the distance from a sited facility to each demand is a global minimum (Jean et al., 2007). The distance measure between a generator and a demand point in Voronoi diagram solution to facility allocation problems or any other field of science and humanitarian services is Euclidean (straight line) and rectilinear distances (Festus and Ernest, 2024).

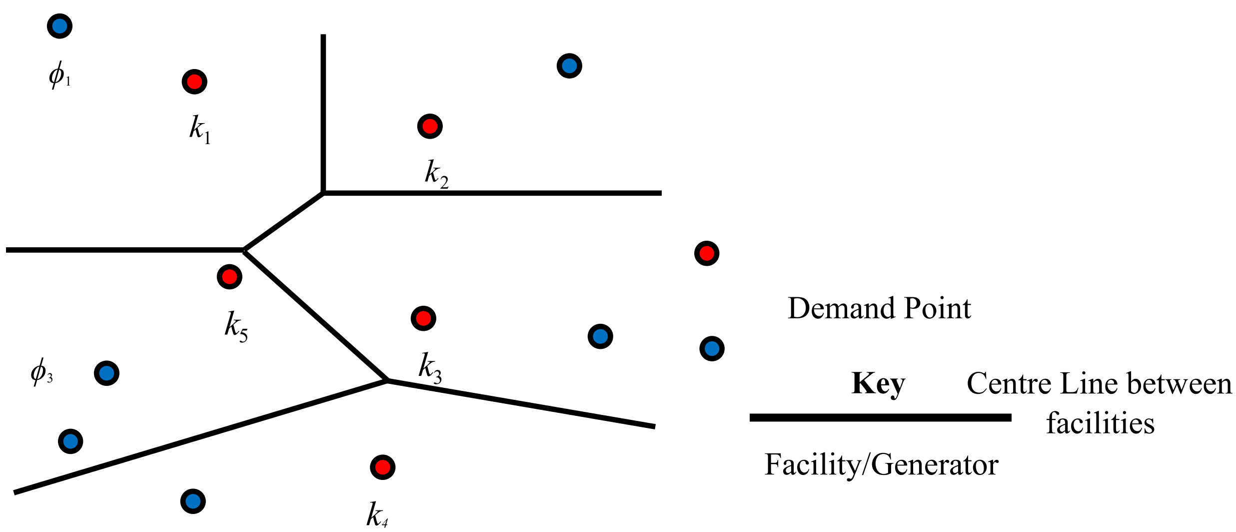

Figure 2.1: Euclidean Voronoi partition of space S containing 5 generators (facilities) and a demand point per facility site

Figure 2.1 Euclidean Voronoi diagram of set **Z** of five generators (sites) and one demand point per site. Consider a set **Z** of an ***n*** number of generating points i.e. $Z = \{k_1,...,k_n\}$ in a space ***S*** representing the service region where the generating points are located in Euclidean space R^2, when the distance $H\emptyset_j K_i$ between a demand point ϕ_i and a generating point k_i is measured or calculated using the Euclidean distance metric. The Voronoi diagram defining such allocation is the sum of the set of the polygon.

$\boldsymbol{V = \{V_{ki},\ldots,V_{kn}\}}$

Where polygon V_{k_i} is given as;

$V_{k_i} = \{\phi_j \subseteq S \,/\, H_{\phi jki} \leq H_{\phi jk'i}\ ,\ \forall_{ki} \in Z\ \&k_i \neq k_i'\}$-- (1)

$\boldsymbol{V_{ki}\ldots,V_{kn}}$ are the Voronoi polygons or cells [2]

Where $\boldsymbol{i}$ is the generating points or facility numbers = **1**, …, $\boldsymbol{n}$ and $\boldsymbol{j}$ are the demand point numbers = **1**, …, $\boldsymbol{m}$. The distance between a generator k and a demand point $\emptyset$ defined on a two-dimensional Cartesian plain with respect to Euclidean distance measure for a single facility and or demand point is given as;

$H_{k\phi} = \sqrt{(x-a)^2 + (y-b)^2}$-- (2)

Where $\boldsymbol{x}$ and $\boldsymbol{y}$ are the coordinates of the generator and $\boldsymbol{a}$ and $\boldsymbol{b}$ are those of the demand point, and $\boldsymbol{H_{k\phi}}$ is the distance between the two points (Jean et al.,2007).

Most often, each of the generating points have weight attached to them to allow for the different traffic rate (Wilfredo et al., 2012).

In some field of studies, the weight attached to a generator (generating point) is a reflection of population density of a neighborhood, the area of facility location, the storage capacity of a source (warehouse) the populating of facility and so on. The attached weight to the generators expands the measured distance in different ways. One of such distance expansion is through the additive weighted distance measure which is stated in equation 3;

$\bar{H}_{k\phi} = H_{k\phi} + \theta_k$ --- (3)

Where $\bar{H}_{k\phi}$ is the resultant distance of adding the weight $\boldsymbol{\theta_k}$ attached to the generating point k to the distance between the generator and the demand point $\boldsymbol{\phi}$.

The second distance expansion measure is a multiplicative weighted distance measure, which is stated as follows;

$\bar{H}_{k\phi} = \theta_k * H_{k\phi}$ --(4)

The framework of the Voronoi diagram runs on the principles of closest assignment, where facilities are placed closest to their demand points through a delineation of a continuous space into various polygon. Figure 2.1 shows a Voronoi diagram partitioning space S into five Voronoi polygons each with a generating point drawn to meet at a mid-point. The boundaries of each polygon are the center lines between each generating point and a demand point.

Suppose there are m numbers of demand points to be served by each generating point i.e. for every $\boldsymbol{k,j = 1 \ldots, m}$, then it will be required for the optimal values of the coordinates of every $\boldsymbol{k}$ to be evaluated with respect to its assigned demand point to achieve the total minimum distance per cell or polygon which subsequently sums up to determine the minimum distance of the entire Voronoi diagram. The distance function per generator (facility) in each polygon which is to be minimized to obtain the optimal values of $\boldsymbol{x_i}$ and $\boldsymbol{y_i}$ is stated in equation 5;

$f\ (x_i, y_i) = \sum_{j=1}^{m} w_i \sqrt{(x_i - a_{ji})^2 + (y_i - b_{ji})^2}$ ---(5)

Where $\boldsymbol{x_i}$ and $\boldsymbol{y_i}$ are the **x** and **y** are the coordinates of the i^{th} generator the objective is to find (x_i^*, y_i^*) that satisfies

$f\ (x_i^*,\ y_i^*) = \min_{x_i\, y_i} f\ (x_i, y_i)$ --(6)

The process of determining the optimal values of the coordinates of each generating facility is that of an iteration of the non-convex function of the weighted total Euclidean distance formed with a multi facility multi demand point problem (Jean et al., 2007). The iteration process starts with a gravity problem model evaluation of the initial values of the x_i and y_i coordinates of the i^{th} facility location, until a convergence is established. The gravity model approach gives the following initial values of x and y with respect to each polygon or facility site.

$$x_o = \frac{\sum_{j=1}^{m} w_i a_{ij}}{\sum_{j=1}^{m} w_{ij}} \quad \text{------} \quad (7)$$

$$y_o = \frac{\sum_{j=1}^{m} w_i b_{ij}}{\sum_{j=1}^{m} w_{ij}} \quad \text{------} \quad (8)$$

Where x_o and y_o are the initial or starting values of the x_i and y_i coordinate of the i^{th} facility (generator), a_{ji} and b_{ji} are the coordinates of the j^{th} demand point of the i^{th} generator (facility) and W_{ij} is the weight of the i^{th} generator (facility) with respect to j^{th} demand point which is the same as W_i x_o and y_o are the initial values of the x_i and y_i coordinate of the i^{th} generator, and m is the number of the demand points per FOB location.

As stated earlier, finding the optimal solution mathematically when using Euclidean distance measure is not easy, hence another distance function is defined that allows the iteration process to continue to obtain the optimal values of x_i and y_i. The said distance function for each polygon or generating point is given as follows;

$$g_{ij}(x_o\, y_o) = \frac{W_i}{\sqrt{(x_o - a_{ij}) + (y_o - b_{ij})}} \quad \text{------} \quad (9)$$

Where $g_{ij}(x_i\, y_i)$ is the distance function of the i^{th} facility or generator with respect to the j^{th} demand point within the facility cell or domain.

The second stage of the Heuristics iteration in the determination of the optimal values of x_i and y_i evaluate i^{th} facility coordinates as follows;

$$x_{ic} = \frac{\sum_{j=1}^{m} a_i g_{ij}(x_i, y_i)}{\sum_{j=1}^{m} g_{ij}(x_i, y_i)} \quad \text{------} (10)$$

$$y_{ic} = \frac{\sum_{j=1}^{m} b_i g_{ij}(x_i, y_i)}{\sum_{j=1}^{m} g_{ij}(x_i, y_i)} \quad \text{------} (11)$$

C is the number of times of iteration

(x_i^*, y_i^*), is achieved by the repetitions of evaluation of eqn. (10 and 11) for every new value of eqn. (9) until a convergence is reached.

The total minimum distance around the entire Voronoi diagram which is the sum of the minimum distance of the various polygons is given as follows;

$$f(x, y) = \sum_{i=1}^{n} \sum_{j=1}^{m} w_i \sqrt{(x_i - a_{ji})^2 + (y_i - b_{ji})^2} \quad \text{------} (12)$$

Where $\boldsymbol{x}$ and $\boldsymbol{y}$ are the sum of the $\boldsymbol{x}$ and $\boldsymbol{y}$ coordinates of each facility round the entire Voronoi diagram.

2.3 Motivational Environment for the Existing Model Application

The motivational environment for the application of the weighted Voronoi diagram is the Nigeria Army Logistic strategy and tactical unit. In the effort of the Nigerian Army to decimate the insurgence in the North Eastern part of Nigeria, the logistics, strategy and tactical unit has introduced the idea of the Forward Operating Base (FOB) to aid in the supply of logistics materials to logistics nodes located within the theatre of operation to attain quick replenishment of materials to the combatant teams. Currently, the conventional mode of the FOBs sitting or location by the Nigerian Army has no defined pattern, hence, some of the logistics nodes are quit a distance from the FOBs thereby increasing resupply time with its attendant effect of higher casualty level on the side of the combat team. It has been observed that the FOBs in some regions are enough to effectively service the logistic nodes at shorter replenishment time but have failed to achieve the objective due to a faulty allocation system that rather clusters the FOBs resulting in appreciable distance to the logistics node. The quest therefore, is that of developing or application of a spatial delineation model to help locate the
FOBs appropriately to achieve shorter replenishment time or distance between the FOBs and the logistics node for higher efficiency and effectiveness of the combat team.

2.4 Propose Model development for Forwards Operating Base Location

The importance of facility allocation models in logistics networking of disaster management, emergency and humanitarian operations, publics utility siting and space delineation exercise cannot be over emphasized. Irrespective of this wide applicability, military operation especially in asymmetric warfare environment is yet to witness its deployment as it relates to facility allocation with respect to demand points. In the course of this model development, the FOBs will be referred to as facilities or generating points interchangeably and the logistics node where the task teams are located will be referred to as demand points.
Figure 2.2 is a map of the North East region of Nigeria with reference to the three states of Yobe, Borno and Adamawa. These states are the worst hit by the activities of insurgence. Figure 3.1 is a grided is a grided map of the North-East region of Nigeria with reference to the three states of Yobe, Bornu and Adamawa which are the worst heat by the activities of insurgence. With particular focus on Bornu state, some locations like Dambowa, Konduga, Bama, Dikwa, Ngala, Maiduguri, Monguno and Magumeri emphasized with bold lines as study areas or sites. The grid system allows the position of a point within the selected area to be located following the direction of its coordinates. Figures 4.2 shows each location with arbitrarily chosen four logistic nodes. The entire space around the eight location is $\boldsymbol{S}$ and $\boldsymbol{Z}$ is the set of **FOBs** $= \boldsymbol{k_1, k_2, k_3, k_4, \ldots, k_8}$.

2.5 Formulating the Distance Function for each Location

Suppose there is one generator (FOB) each in every of the eight locations of Dambowa, Konduga, Bama, Dikwa, Ngala, Maiduguri, Monguno and Magumeri and labelled k_1 to k_8 respectively i.e. $i = 1, \ldots, 8$ and the number of demand points per location are four for $j = 1, \ldots, 4$. Then we can write the distance function for each of the eight locations with respect to their demand points as generated from eqn. 5 as follows;

$$f(x_i, y_i) = \max_{\forall i}\left(\sum_{j=1}^{m} w_i \sqrt{(x_i - a_{ji})^2 + (y_i - b_{ji})^2}\right) \text{ -- } (13)$$

Where ***i* = 1, …, 8, *Wᵢ*** is the weight attached to the FOB in each location. The weight in this study is a reflection of the degree of difficulty in accessing each location. From figure 3.1 and 3.2, the hatched areas show locations with difficulty in accessing. Those areas include Dambowa, Bama, Dikwa, Ngala and Mongumu, hence will be assigned higher traffic weight than those of Maiduguri, Konduga and Magumeri that shows a topography with easy access.
The objective is to find **(x_i, y_i)** that will satisfy

$$f(x_i^*,\ y_i^*) = \min_{\forall i} f(x_i, y_i) \text{ --- (14)}$$

It can be inferred from eqn. (3.12) that the total Euclidean distance around the eight sites will take the form'

$$f(x, y) = \sum_{i=1}^{8} \sum_{j=1}^{4} w_i \sqrt{(x_i - a_{ji})^2 + (y_i - b_{ji})^2} \text{ -- (15)}$$

The objective is to find $(x_i^*,\ y_i^*)$ that satisfy

$$f(x_i^*, y_i^*) = \min_{(x_i,\ y_i)} f(x_i, y_i) \text{ -- (16)}$$

The process of satisfying the above objective of EQN (15) starts with that of minimizing eqn. (13). Four demand points are chosen arbitrarily and located at the geographical (spatial) boundaries of each location. The iteration process of eqn. (7), (8), (9), (10) and (11) is carried out in each location with respect to the coordinates of the selected field demand points (logistic nodes).
After each iteration, a new Voronoi diagram emerges consequent on the current FOBs coordinates. An optimal configuration is attained i.e. the values of the FOBs coordinates are the same or with no significant different at each iteration. At the optimal iteration of the FOBs a center line is drawn between each FOBs site to intercept at a meet point for all the FOBs at this stage, the entire ***S*** space embedding the eight locations will be partitioned by the boundary lines of the Voronoi polygon (cells) that will emerge from the joining of the several center lines. The Voronoi diagram partitioning, may allocate some logistic nodes in one location to other locations depending on their current state of closeness to any FOB irrespective of its parents' location. This outcome will appropriately suite the aim of this study by locating each demand point to each designated location on the bases of closest assignment. With the shortest distance between FOB and demand points achieved through better logistic network orientation, cost, combat efficiency and other distance related factors will be optimized.

2.6 Operational Cost Formulation

Operational success may have different definition in each field of human studies with respect to the variable of significance that requires optimization. In commercial supply chains, optimal Voronoi diagram configuration is attained when total transportation cost of supply of goods to all the demand points in the region around the service Domain is minimum Willfredo et al, (2012). In asymmetric or unconventional warfare scenario where the target is minimizing casualty level arising from surprise attack (ambush) unfriendly topography, difficulty in resupply of logistic materials and other sundry issues, operational success may be defined differently. Here, the minimization of resupply or response time which is the major component of the engagement cost function formulated to represent the war dynamics is the underlining measure of success (Veras et al. 2005: Akkihal, 2007) The operational cost to be formulated in this study could be related to the deprivation cost presented in the studies of (Wilfredo et al., 2012; Veras et al., 2005) which they stated as follows

$f(\Delta t) = p_i e^{(\alpha+\beta\Delta t)}$ -- (17)

Where $\boldsymbol{\Delta t}$ is the time since the last resupply, $\boldsymbol{e}$ is the mathematical constant for inverse natural logarithm and $\boldsymbol{\alpha}$**,** $\boldsymbol{\beta}$ are parameters calibrated using some statistical tools from the presented economic data and p_i is the population of the $\boldsymbol{i^{th}}$ location. They suggested that the longer the resupply time, the more deprived are the individuals who need aid in the affected region of disaster in the relief management chain. Similarly, in this study, we shall adopt the same approach to develop an engagement cost function for the $\boldsymbol{i^{th}}$ FOB, except that, $\boldsymbol{p_i}$ which is the equivalent of $\boldsymbol{w_i}$ in our study will be omitted since the Euclidean distance measure is the weighted type. Through statistical methods, the parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ will be determined from the military engagement data obtained. Two new terms, $\boldsymbol{P(A)_{ij}}$ and $\boldsymbol{(C_A)}$ which are the probability of an ambush between the $\boldsymbol{i^{th}}$ FOB and the $\boldsymbol{j^{th}}$ demand point and the ambush casualty value will be introduced into the formulated engagement cost function. The expression of the engagement cost is as follows:

$f(\Delta t, C_A) = e^{\alpha + \beta\Delta t} + e^{[(p(A)_{ij}) \cdot (CA)_i]}$ -- (18)

In eqn (18), the variable $\boldsymbol{P(A)_{ij}}$ and $\boldsymbol{\Delta t}$ are very vital in defining the operational success of the war of the Nigeria army against insurgence. Winning the war will mean total decimation of the insurgence with near zero casualties on the side of the Nigeria army. A tactical requirement to achieve this is to locate the logistics nodes close enough as possible to each designated FOB to allow for faster replenishment of expendable and non-expendable needs of the combat team. This time or distance-based strategy when optimally developed will not only act as moral booster to the combat team, but will allow for sustained assault and eventual deflation of the adversary momentum. The second term which is the possibility of an ambush by the enemy though not a time factor but majorly that of topography, could erode the gains of the distance-based factor if not properly checked. Its relationship with distance is the delay on resupply as the replenishment network is momentarily halted by its occurrence. Since this study is not devoted to analysis of the procured superior hardware capable of neutralizing ambushes, we develop a cost function that accommodates it to show its impact on engagement cost.

Assuming a time-distance proportionality, where greater distance represents a larger resupply time, then the time difference expression **(**$\boldsymbol{\Delta t}$**)** of eqn. (3.18) can be put in term of distance H, thereby defining a distance function $f(H)$ (Veras et al.,2006). This implies that, an $\boldsymbol{i^{th}}$ FOB that is seeking to minimize engagement cost within its domain must like to min$[f_i(H_i)]$ (Darusz, 2016). Where $\boldsymbol{H_i}$ represent the minimum weighted Euclidean distance between the $\boldsymbol{i^{th}}$ FOB and its demand point within the $\boldsymbol{V_{ki}}$ polygon.

Suppose $\boldsymbol{f(k)}$ represents the total engagement cost incurred around the service region of the eight FOB locations, provided all the FOBs are located in K, the engagement cost function can be expressed as follows;

$f(k) = \sum_{i=1}^{8}\sum_{j=1}^{4} w_i\sqrt{(x_i - a_{ji})^2 + (y_i - b_{ji})^2}$ -- (19)

The objective is to find $\boldsymbol{f(k^*) = Min\, f(k)}$. Again, the weighted Euclidean distance between the $\boldsymbol{i^{th}}$ facility (FOB) an the j^{th} demand point which is H_{ij} is given as

$H_{ij} = \sum_{j=1}^{4} w_i\sqrt{(x_i - a_{ji})^2 + (y_i - b_{ji})^2}$ -- (20)

Let $f(H_{ij})$ be a distance function, such that;

$$f(H_{ij}) = \alpha + \beta(H_{ij}) \quad (21)$$

It follows that the total engagement cost of eqn. 3.19 can be rewritten as:

$$f(k) = \sum_{i=1}^{8}\sum_{j=1}^{4} e^{f(H_{ij})} + e^{p(A)_{ij}(CA)_{ij}} \quad (22)$$

Where $P(A)_{ij}$ is the probability of an ambush in the $\boldsymbol{V_{ki}}$ polygon and $(CA)_{ij}$ is the ambush casualty value. The addition, $e^{p(A)_{ij}(CA)_{ij}}$ in eqn (3.22) is the portion that explains the influence of enemy's successful ambush on the operational cost of the military. It underscores the essence of modern technology in exposing and detecting ambushes especially in asymmetric warfare.

A successful ambush whether partial or total with respect to what logistics materials, hardware and personnel recovered after the ambush will certainly affect the resupply times, which will further increase the casualty level at the theatre of operation if a contingent plan is not put in place.

3. RESULTS AND DISCUSSION

3.1 Validation of the Voronoi diagram location of FOBs and Determination of Relevant measures

In the sequel, the validation of the Voronoi diagram allocation of the forward operating bases (FOB) and the determination of relevant measures are presented

3.1.1 Validation of the proposed Voronoi diagram location of FOBs in North east Nigeria

In the validation of the proposed allocation of supper camps or forward operating bases, in the north east region of Nigeria particularly in Borno State being the study case, primary and secondary data were obtained from the Nigeria army headquarters, structural interviews and questionnaires and internet publications.

Eight locations from Borno State where the Nigeria army currently have FOBs were selected. These locations are, Dambuwa, Konduga, Bama, Dikwa, Ngala, Maiduguri, Monguno and Magumeri. Secondary data from the Nigeria army records (classified) shows that there is more than one FOB in each location especially in Bama where about five FOBs are located. It is observed that with the more than one FOB in each location, the demand points or logistics nodes to be served by the FOBs are too far apart, some in the range of 30-40km, when the original design objective is to attain a distance of at last 15km apart. Again, because of a non-scientific allocation model, FOBs in some locations are clustered around a point within the intended service domain, hence, all the FOBs fail equally in the advantage of short resupply time and distant demand or logistics points. We shall begin this validation process by assigning four arbitrary demand points to the chosen locations and mathematically analyzing coordinates to determine the optimal coordinates of one FOB per location. This is to enable us partition the considered space covering the eight selected locations into several cells or polygons to FOB to ascertain which FOB can best serve which demand point. We shall then establish by comparison in terms of nearness of demand points to the evaluated FOB position and that of the army's conventional assignment policy. Table 4.1 shows the abscissa and ordinates of four arbitrarily chosen demand points per each of these eight selected locations for analysis in Borno State. The blue spots per location in Figure 4.1. (Map of Borno State drawn to scale) represents the demand point).

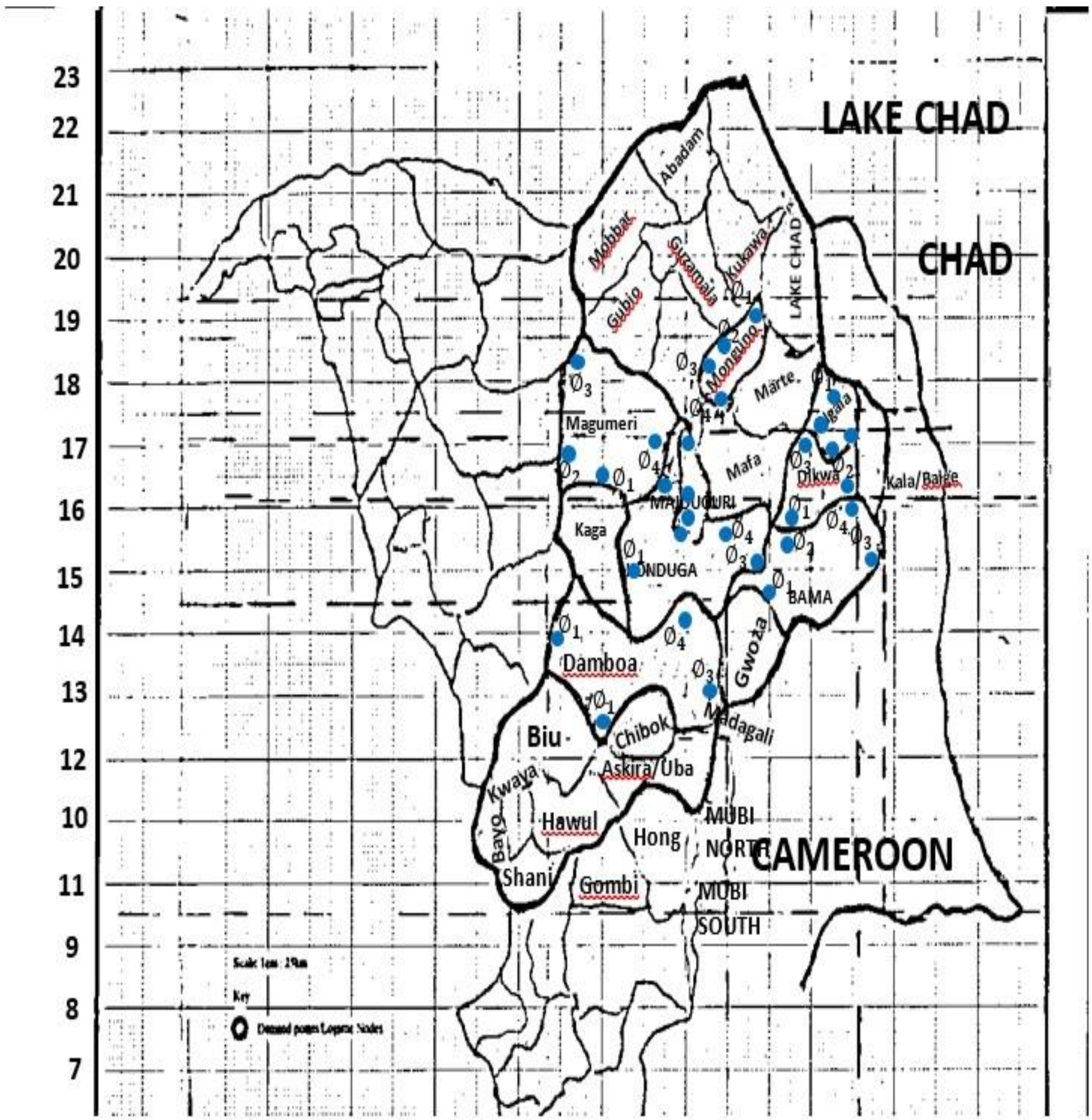

Figure 3.1: Map of Borno State with four (4) Demanding points (Logistic nodes) per the eight selected locations

Table 3.1: The abscissa (a) and ordinate (b) of the four Arbitrary chosen Demand point per location

Location	a - coordinates per demand point (cm)				b - coordinates per Demand point (cm)			
Dambowa	9	10	12.4	12	14	12.4	13	14
Konduga	11	11.5	13.7	13	15	16.2	15	15.5
Bama	14	14.5	16.5	14	14.5	15.3	15	15.9
Dikwa	15	15.9	14.5	14.8	16	16.4	15.3	16.8
Ngala	15	15.3	15.4	15.9	17.8	17.3	16.8	17
Maiduguri	12	12.4	12	12	15.4	15.7	14	14.8
Monguro	13.6	13	12	12.8	19	18.5	18	17.5
Magmeri	10	9.1	9.2	11.3	16.4	17	18	17

Coordinates a and b adopted in this study is to differentiate between the coordinates of the demand point and that of the **x** and **y** for the FOBs.
From table 3.1: Each location is assigned four arbitrary demand points with coordinates as stated below in **cm**
Dambowa = (9,14)(10,12.4),(12.4, 13) (12.14) (11.5, 14.2)
Konduga = (11,15) (15.5, 16.2) (13.7, 15) (13.15.5)
Bama = (14, 14.5) (14.5, 15.3) (16.5, 15) (14.15.9)
Dikwa = (15,14) (15.9, 16.4) (15.5, 15.3) (14.8, 16.8)
Ngala = (15.5 17.8) (15.3, 17.3) (15.4, 16.8) (15.9, 17)
Maiduguri = (12, 15.4) (12.4, 15.2) (12,16) (12.16.8)
Monguno = (13.6, 19) (13, 18.5) (12.4, 18) (12.8, 17.5)
In each of those locations, an initial value of x_o and y_o is set for an FOB position using eqn 7 & 8. The iteration process then continues until optimal values of x^* y^* are obtained. Given that it is desired that one FOB be sited per location to allow for effective comparison, we shall allow i, (the number assigned per FOB location) to run from 1 to 8 with respect to the listing of the locations above and also, holds that, (the number of demand point assigned per location) runs from 1 to 4 for the demand points per location
Recall eqn (7),

$$x_o = \frac{\sum_{j=1}^{m} w_i a_{ij}}{\sum_{j=1}^{m} w_{ij}}$$

$$y_o = \frac{\sum_{j=1}^{m} w_i b_{ij}}{\sum_{j=1}^{m} w_{ij}}$$

Dambowa is difficult to assess from the information provided, it is assigned a weight value of $\mathbf{W_1 = 5}$. In the same vein Bama W_3, Dikwa W_4, Ngala W_5, and Mongouno W_7 are all assigned weight 5. Similarly, Konduga W_2, Maiduguri W_6, Magumeri W_8, are assigned weight 2 for their easy access.

Note $\boldsymbol{w_{11} = w_{12} = w_{13} = w_{14} = 5}$ since it is a movement within the same location

For Dambowa,

$$x_o = \frac{\alpha_1 \times w_1 + \alpha_2 w_1 + \alpha_3 w_1 + \alpha_4 w_1}{w_{11} + w_{12} + w_{13} + w_{14}}$$

$$x_o = \frac{9 \times 5 + 10 \times 5 + 12.4 \times 5 + 12 \times 5}{20} = \frac{217}{20} \approx 10.9 \text{ cm}$$

Similarly

$$y_o = \frac{b_1 w_1 + b_2 w_1 + b_3 w_1 + b_4 w_1}{w_{11} + w_{12} + w_{13} + w_{14}}$$

$$y_o = \frac{14 \times 5 + 12.4 \times 5 + 13 \times 5 + 14 \times 5}{20} = \frac{267}{20} = 13.4 \text{ cm}$$

Hence, the initial coordinates of the FOB in Dambowa are (10.9, 13.4).

Applying equation 3.9 and following the iteration process yields optimal coordinate for the FOB in Dambowa with respect to its demand points as;

$(x_1^*, y_1^*) = (11.0,13.3)$

The optimal coordinates for the other locations are given as follows in cm;

KONDNGA $(x_2^*, y_2^*) = (12.0,15.4)$

BAMA $(x_3^*, y_3^*) = (15.3,15.3)$

DIKWA $(x_4^*, y_4^*) = (150,16.1)$

NGALA $(x_5^*, y_5^*) = (15.5,17.2)$

MAIDUGURI $(x_6^*, y_6^*) = (12.1,15.8)$

MONGUNO $(x_7^*, y_7^*) = (13,18.2)$

MAGUMERI $(x_8^*, y_8^*) = (9.9,17.0)$

Given that the optimal coordinates of each FOB in every location have been evaluated as shown above, they are carefully sited in each location using the grid markings and represented with, red spot in figure 4.2. Applying the center line and triangulation method discussed earlier (Grzesica 2016), the entire space embedding the eight locations is partitioned into eight cells or polygons as shown in figure 4.3. The Voronoi diagram generated in figure 4.2 formed by the joining of the straight thick boarder liens obtained by the joining of the center lines between each FOB also separate same from each other and allows demand points best served on the bases of nearness to the enclosed demand point in the same space with such FOB. The further the demand point from a designed FOB to serve same the higher the value of the casualties of war occasioned by delay in the re-supply of the necessary materials to prosecute the war. Table 4.2 shows the changes in the engagement causality levels alongside the changing distance of the demand points to the FOB location.

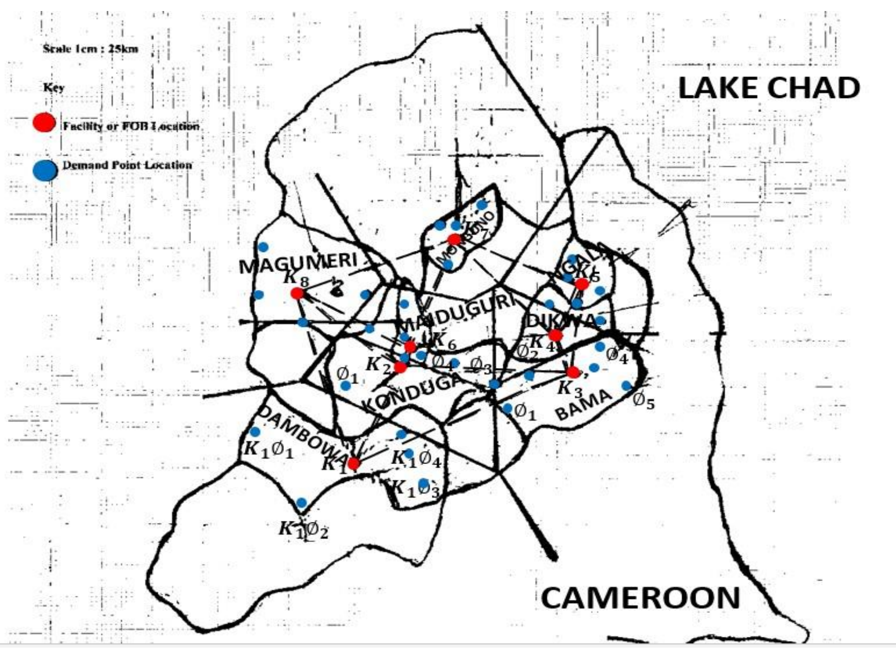

Figure 3.2: The Construction of a Voronoi Diagram for the eight selected location of Borno State

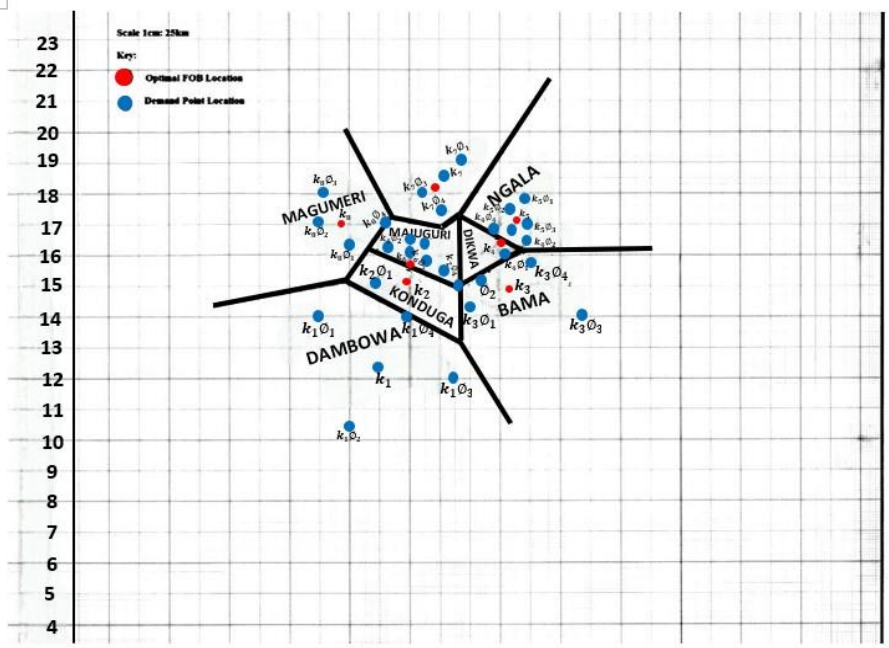

Figure 3.3: An Optimal Voronoi partitioning of the space embedding the eight selected locations of Borno State

Table 3.2: Engagement casualty level (C_{eL}) for varying distance of demand points from FOBs in BAMA

Engagement casualty level C_{eL}	**Distance of demand point to FOB d(km)**	**Combat Team Size (persons)**
5	0	
10	5	
14	10	
18	15	
22	20	
28	25	100
31	35	
37	30	
43	40	
49	45	
52	50	
65	55	

Source: Primary Data from Administered Questionnaires to Ex-Combat Team members.

Determination of the α and β parameters for C_{eL} and d values

The values of the engagement casualty level C_{eL} from the operation in Bama against the changing distances (d) between demand points and FOB in Table 3.2 do not follow any pattern. This is primarily because the respondents who principally were casualties themselves gave answers of overrated casualty levels, due to depravation suffered hence,

became emotional and gave answers that intend to stir up sentiment. Accordingly, we intend to evaluate an estimator of C_{eL} i.e.
C'_{eL} for every distance d since C_{eL} is the dependent variable
From regression theory, we recall that,

$$C'_{eL} = \alpha + \beta d \quad (23)$$

Where α and β are the parameters of the regression equation

TABLES 3.3: Determination of the values for **C_{eL}** ***and*** **d** ***for regression equation***

C_{eL}	d	$C_{eL} * d$	d^2	C^2_{eL}
5	0	0	0	25
10	5	50	25	100
14	10	140	100	194
18	15	270	225	324
22	20	440	400	484
28	25	700	625	784
31	30	930	900	961
37	35	1295	1,225	1,369
43	40	1720	1,400	1,849
49	45	2205	2,025	2,401
52	50	2400	2,500	2,704
65	55	3575	3,025	4,225
374	330	13,925	12,650	15,422

$$\beta = \frac{n\sum dC_{eL} - (\sum d)(\sum C_{eL})}{n(\sum d^2) - (\sum d)^2}$$

Where n, the number of observations = 12

$$\beta = \frac{12(13,925) - (330)(374)}{12(12,650) - (330)^2}$$

$$= \frac{43,680}{42,900} = 1.0182$$

Again

$$\alpha = \frac{\sum C_{eL}}{n} - \beta\left[\frac{\sum d}{n}\right.$$

$$= \frac{374}{12} - 1.0182\ \frac{330}{12} = 31.167 - 28.0005$$

$= 3.167$

$= C_{eL} = 3.167 + 1.0182d$

For d=0, $C'_{eL} = 3.167 = 3$

Clearly, C'_{eL} is a good estimator of C_{eL}, and has produced values that are more regular in pattern

Table 3.4: The values of C'_{el} *and* d *values of Bama location*

C_{eL}	3	8	13	18	14	29	34	39	44	49	54	59

d(km)	0	5	10	15	20	25	30	35	40	45	50	55

Table 3.5: Observed number of ambushes per 50 trips between demand points and FOB in Bama location

3	0	0	1	4	2	1	3	1	0
2	0	1	0	0	5	1	1	0	0
0	4	5	1	2	0	1	1	4	0
3	2	3	0	1	3	0	3	2	0
1	0	2	1	0	4	3	5	1	0

Source: Primary Data; Results of Questionnaires Administered on Ex-Team Members

Determination of the expected number of ambushes per 50 trips in Bama location

The probability of the occurrence of any value $p(a)$ could be expressed as follows; ss

$$p(a) = \frac{f}{P}$$

Considering the entries in table 4.5, for a value of 0, f=17, p=50, hence the probability of o occurring for the 50 trips is given by

$$p(a = 0) = \frac{17}{50} = 0.34$$

Recall that $U_m = \sum[a.p\,(a)]$

$= 0 + 0.26 + 0.24 + 0.42 + 0.32 + 0.2 = 1.44$

Since $\boldsymbol{U_m}$ represents the expected number of times an ambush will occur in 50 trips between any demand point and an FOB within BAMA location, we shall estimate the probability of Ambush in BAMA to be $p(A) = \frac{U_m}{50} = \frac{1.44}{50} = 0.03$

Areas like BAMA with difficult topography and bad terrain as earlier stated are prone to ambushes, save for modern military hard wares that have reduced the successes of such operations by the insurgences. The heavy traffic weight of 5 imposed on this location from previous evaluation underscores the relevant of this assessment issue. Casualty levels from ambush in the difficult terrain from military records though classified are either fatal with all death and all destruction, partial with death and injured or less fatal with mild injuries and scratches on hard wears. Nevertheless, we shall assume a casualty level of 10 (persons and equipment) whenever it occurs since the classified nature makes it impossible to assess from the military records.

Determination of the engagement cost $f(k_3)$ for Bama Polygon (cell) in fig 3.3

Recall that the total engagement cost function for the entire eight selected locations as given in eqn. 22 was as follows;

$F(k_i) = \sum_{i=1}^{8}\sum_{j=1}^{4} e^{f(H_{ij})} + e^{p(A)_{ij}(CA)_{ij}}$ where $f\,(H_{ij}) = \alpha+\beta(H_{ij})$, H_{ij} is the Euclidean distance between the i^{th} FOB and the j^{th} demand point, and C_A is the ambush casualty value and $P(A)_{ij}$ is the ambush probability in the course of replenishment of the demand points within the eight locations of the i^{th} FOB. The relaxed version of eqn.22 for the individual polygons is given as

$$F(k_3) = \sum_{j=1}^{4} e^{f(H_{ij})} + e^{p(A)_{ij}(CA)_{ij}} \quad \text{for } i = 3$$

Where the number of the FOB in the third location, i.e. Bama is ***i* = 3.** It follows that;

$$f(H_{ij}) = \alpha + \beta H_{ij} = 3.167 + 1.0128H_{ij}$$
$$Where\ H_{ij} = \sqrt{(x_3{}^* - a_3)^2 + (y_3{}^* - b_3)^2}$$

H_{31} = 1.5cm
H_{32} = 0.8cm
H_{33} = 1.2cm
H_{34} = 0.9cm
$P(A)_{31}$ = 0.03
$f(H)_{31} \approx$ 5cm
$f(H)_{32} \approx 4cm$
$f(H)_{33} \approx 4cm$
$f(H)_{34} \approx 4cm$

$f(k_3)$ = 317.6

The above expressed costs are in terms of causality values, hence will carry a monetary value if the unit causality cost is estimated or provided by the Nigeria Army with respect to how much is spent on a deceased person or the treatment cost and equipment recovery cost.

Analysis of the Nigerian Army conventional location of FOBs.
The Nigerian Army conventional location of FOBs allows for several. FOBs to be located in a region to service different demand point as the need arises. Contrary to the scientific model that we have explored, where the optimal coordinates of the FOBs with respect to those of the demand points are determined to minimized distances, it is necessary to establish if there is a pattern to that of military allocation and if there an attempt to minimize distance.
Table 3.6 shows the longitudinal and latitudinal positions of the eight selected locations in Borno State and the five (5) FOBs allocated to Bama.

TABLE 3.6: LONGITUDINAL AND LATITUDINAL POSITION OF THE EIGHT SELECTED LOCATIONS IN BORNO STATE

Locations	Latitude	Longitude	FOBS AND THEIR PAIRED CORIDNATES
DAMBOWA	11°09' 19.22" N	12°45' 37.46" E	

KONDUGA	11^{O}39’ 3.59’N	13^{O}25’ 5.99” E					
BAMA	11^{O}31’ 16.82” N	13^{O}41’ 22.27” E	K_{35} (14, 13)	K_{34} (13.8 13.9)	K_{33} (15, 13.1)	K_{31} (14.8 14)	K_{32} (14.8, 14.4)
DIKWA	12^{O}02’ 9.92” N	13^{O}55’ 5.34” E					
NGALA	12^{O}20’ 25.91” N	14^{O}11’ 12.12” E					
MADUGURI	11^{O}49’59.99” N	13^{O}9’ 3.48” E					
MONKUWO	12^{O}40’12.86N	13^{O}36’ 42.26’’ E					
MAGUMERI	12^{O}6’ 47.42” N	12^{O}49’ 37.34’’E					

Where k_{35} k_{34} k_{33} k_{31} and k_{32} are the five FOBs located in Bama the third locations.

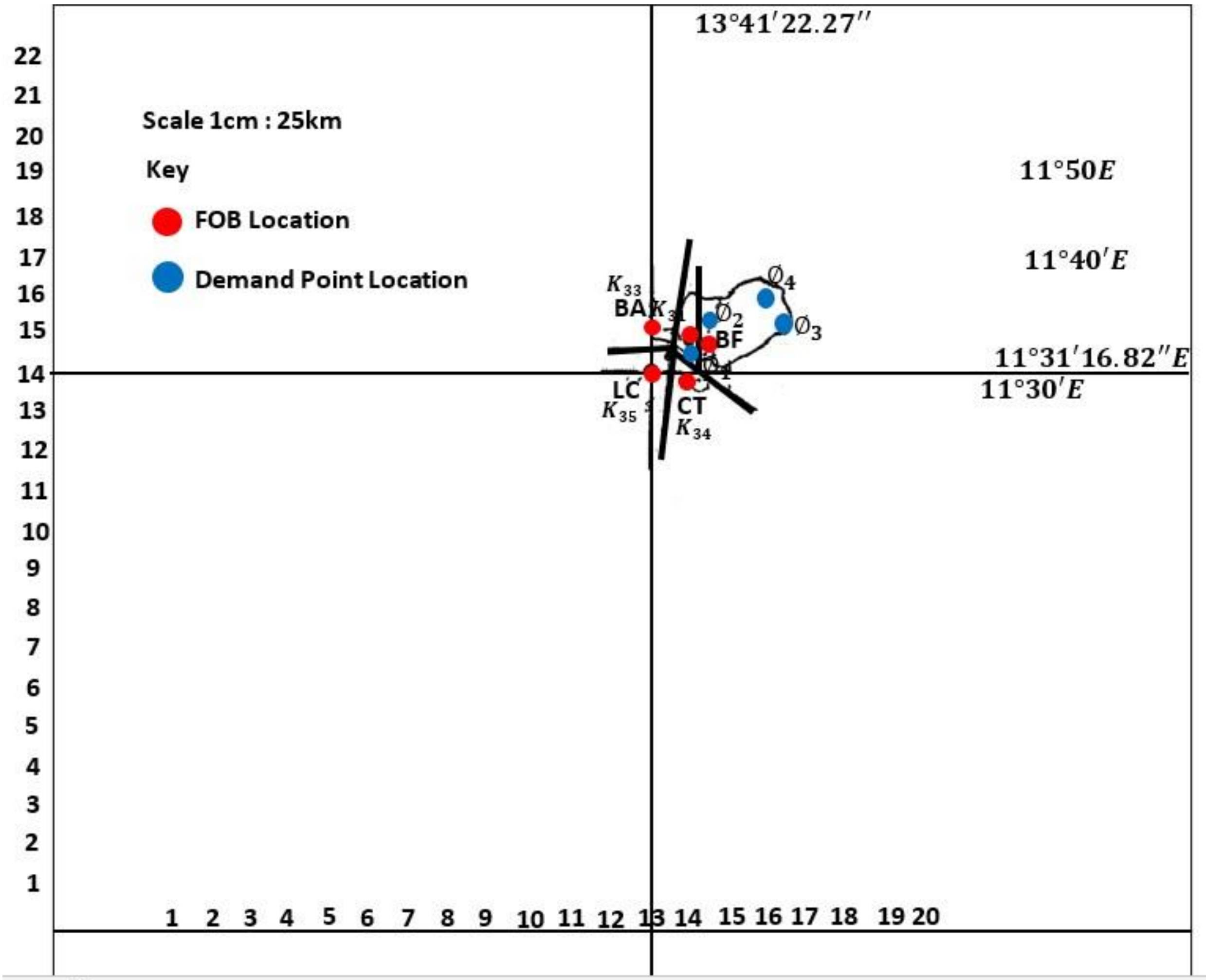

Fig 3.4: A graph of the conventional Nigeria Military Placement of FOBs around BAMA Location

The graph of figure 4.4 shows the locations of the five (5) FOBs in Bama and those of the four (4) demand points chosen in the analysis of the developed model.

In the analysis of the conventional Nigeria military placement of FOBs, a new parameter will be introduced to represent the numbers of location of the FOB placements in Borno State. Hence r_{ij} means, r^{th} location with the l^{th} FOB and the j^{th} demand points.

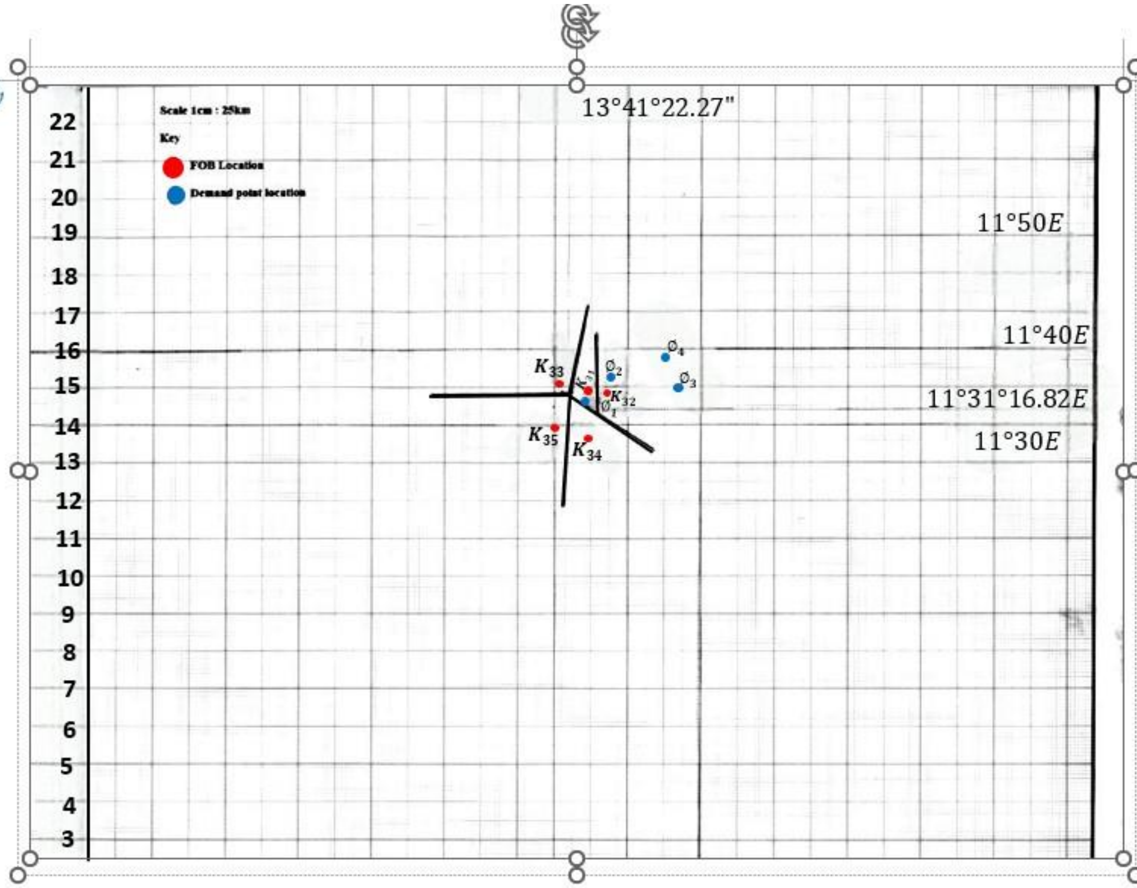

Figure 3.5: Voronoi partitioning into service domain of the space around five (5) FOB locations in Bama

Determination of the distance between FOB (k_{31} and k_{32})

It is obvious from figure 3.4 that only two FOBs (K_{31} and K_{32}) are within the enclosed map of Bama, hence can better service the demand points. Figure 3.5 shows the Voronoi delineation of the space around the five FOBs and diagrammatically excludes K_{33}, K_{34} and K_{35} from the space around the demand points.

From figure 3.5. The paired distance coordinate value of FOB (K_{31}) and FOB (K_{32}) in Bama location by the military allocation is given as

FOB(K_{31}) = (14.8, 14), FOB (K_{32}) = (14.8, 14.4) for the x and y coordinates respectively. The Euclidean distance between these FOBs can be estimated from eqn(2)

$$H_{k31k32} = \sqrt{(x - a)^2 + (y1 - y2)^2}$$

Where H_{k31k32} is the Euclidean distance between the first FOB (k_{31}) and the second FOB (k_{32}) in Bama x_1 and y_1 are the coordinates of K_{31} and x_2 and y_2 and those of FOB k_{32}

$H_{k31k32} = \sqrt{(x - a)^2 + (y1 - y2)^2} = 0.4cm = 10km$ (ground measurement) since 1cm = 25km from the scale of figure 3.4.

Determination of the Euclidean distance between FOB (K_{31} and K_{32}) and the four arbitrary chosen demand points in Bama

From figure 3.4, the four arbitrary, chosen demand pints that were used in the analysis of the proposed model have again been placed in their respective positions to allow for comparison of the proposed model with the conventional military allocation strategy.

The demand points represented with blue spots in fig 4 are listed as $\varphi_1, \varphi_2, \varphi_3, \varphi_4$. Their coordinates as stated earlier are given below

$\varphi_1 = (14, 14.5)$, $\varphi_2 = (14.5, 15.3)$, $\varphi_3 = (16.5, 15)$, $\varphi_4 = (16, 15.9)$

From figure 4.5. Only two FOBs (k_{31}&k_{32}) polygon contains in the demand point by the Voronoi partitioning. The paired coordinates of these FOBs are:

$k_{31} = (14, 14.8)$ ***and*** $k_{32} = (14.4, 14.8)$

The Euclidean distance between $k_{31}$and the respective demands points of $\varphi_1\varphi_2\varphi_3$ and φ_4 is given as:

$H_{k31\varphi1} = 0.3cm$
$H_{k31\varphi2} = 0.7cm$
$H_{k31\varphi3} = 2.5$cm
$H_{k31\varphi4} = 2.6cm$

Where k_{31} is the first FOB in Bama (location 3), and k_{32} is the second FOB in Bama (location 3) and $\varphi_1, \varphi_2, \varphi_3$ and φ_4 are the respective demand points **(j=1..,4)** Similarly, the Euclidean distance between FOB **(k_{32})** and assigned four (4) demand point are as follows

$H_{k32\varphi1} = 0.5cm$
$H_{k32\varphi2} = 0.5cm$
$H_{k32\varphi3} = 2.1cm$
$H_{k32\varphi4} = 1.9cm$

Table 3.7: Distance between FOB k_{31}, k_{32} and demand points φ_1, φ_2, φ_3 and φ_4

	φ_1	Km	φ_2	Km	φ_3	Km	φ_4	km
H31	0.3	7.5	0.70	6.25	2.5	62.5	2.6	65
H32	0.5	12.5	0.5	12.5	2.1	52.5	1.9	47.5

Determination of the engagement cost $f(k_{31})$ and $f(k_{32})$ for BAMA location in figure 3.5.

The engagement cost function of FOB k_{31} with respect to the four chosen demands. Points, φ_1, φ_2, φ_3 and $\varphi_4$in figure 3.5 can be estimated from the relaxed version of eqn 22 as follows:

$f(k_{31}) = \sum_{j=1}^{4} e^{f(H_{ij})} + e^{p(A)_{ij}(CA)_{ij}}$ for $j = 1., \ldots, 4$

$f(H_{31\varphi1}) \approx 4$cm
$f(H_{31\varphi2}) = 4$cm
$f(H_{31\varphi3}) \approx 6$cm
$f(H_{31\varphi4}) \approx 6$cm

Hence, the engagement cost function for the first FOB (k_{31}) in the third location Bama is given as;

$f(k_{31}) = 921.5$cm

Similarly, the distance function of the second FOB $\boldsymbol{f(H_{32\varphi j})}$ for Bama with respect to the four demand points is as follows:

$f(H_{32\varphi1}) \approx 4$cm

$f(H_{32\varphi 2}) \approx 4$cm
$f(H_{32\varphi 3}) \approx 5$cm
$f(H_{32\varphi 4}) \approx 5$cm
Therefore, the engagement cost functions of the second FOB ($f(k_{32})$) for the third location with respect to the four demand points is as follows
$f(k_{32\varphi j}) = 411.43$cm Where j=1,..,4

Table 3.8: Comparison between the distance of the FOBs is demand points and their engagement cost function for the proposed model and that of the military

	φ_1		φ_2		φ_3		φ_4	
	Cm	km	Cm	km	cm	Km	Cm	K
H3	1.5	37.5	0.8	20	1.2	30	0.9	22.5
H31	0.3	7.5	0.3	7.5	2.5	62.5	2.6	65
H32	0.5	12.5	0.5	12.5	2.1	52.5	1.9	47.5
$f(k_3)$		317.4						
$f(k_1)$					921.5			
$f(k_2)$		505.2						

4. Discussion of Results

In this study, eight locations have been selected in Borno State, North eastern Nigeria where the Nigeria Military have their super camps (FOBs) with a design to supply the operational demand points close to the theatre expendable and other logistic materials to win the war against insurgence. In the allocation of the FOBs, which is predicated on shortness of distance as a definition of operational success, a scientific model has been adopted as a more efficient and effective allocation strategy supposedly better than the military conventional allocation techniques. In the development of the model (Voronoi diagram), four arbitrary demand point sites were chosen per location and were analyzed to help obtain an optimal FOB location for each of the selected areas. Fig 3.3 shows how the implementation of this model has helped to delineate the entire space embedding the eight selected locations into eight cells (polygon) with each FOB per locations servicing the closest demand points to it. Considering Bama location as a major reference point, since the military records shows that it has at least five FOBs, we wish to determine whether by principles of closest allocation, if the FOBs are appropriately sited with respect to the demand points within their service domain. The four-demand point (blue dot in figure 3.3) in Bama of $\boldsymbol{\varphi_1}$**,** $\boldsymbol{\varphi_2}$**,** $\boldsymbol{\varphi_3}$ and $\boldsymbol{\varphi_4}$ can best be serviced by the FOB K_3 in Bama. One of the four demands points of

Konduga ($K_2\phi_1$) and one of Maiduguri ($K_6\phi_1$) are better serviced by the FOB in Konduga, while two of the demand points in Konduga ($K_2\phi_2$) and ($K_2\phi_4$) alone side three of Maiduguri ($K_6\phi_2$), ($K_6\phi_3$) *and* ($K_6\phi_4$) are better serviced from the FOB in Maiduguri. The fourth point ($K_2\phi_3$), on the border line between Konduga and Bama can be serviced equally from the FOBs in each location. The FOB in Ngala will better service two of the demand points ($K_4\phi_2$), and ($K_4\phi_4$), in Dikwa, just as Magumeri and Monguro FOBs will service better their demand points. The longest distance between any of the four demand points for the proposed model application in Bama and its FOB in table 3.8, is $H_3\varphi_1 = 1.5cm$

Since the scale on the map is 1cm: 25km, this distance will be
$H_3\varphi_1 = 1.5\times25 = 37.5km$. The shortest distance is, $H_3\varphi_2 = 0.8cm = 20km$. The other distances of $H_3\varphi_2$ *and* $H_3\varphi_4$ are
$H_3\varphi_3 = 1.2m = 30km$ *and* $H_3\varphi_4 = 0.9cm = 22.5km$
The engagement cost function of operating in Bama with respect to the four demand points is, $f(k_3) = 317.9$
Supposing the casualty cost for an individual is NY and there are Z causalities over some defined period, then the engagement cost with respect to casualties will be;
$F(k_3) = N317.9C_c$
Where $C_c = Y.Z$ = the casualty cost per the number of persons injured or killed in an operation.
Discussion of Results of the Convention Military Allocation
Table 3.6 shows the longitudinal and latitudinal location of each of the selected locations in Borno State, Nigeria. Locating these lines of latitude and longitude on a gridded paper and positioning the five FOBs in their respective position alongside maintaining the locations of the demand points as in the case of the propose model will result in figure 3.4. Figure 3.4 represents the conventional allocation of
FOBs by the Nigeria military in Bama location. From the diagram of the Voronoi delineation of figure 3.5, three FOBs K_{33}, k_{34} and k_{35} are outside the map of the
Bama location or at best within the periphery of the location, with their service domain within their delineated boundaries as shown by the black border lines in figure 3.5. From this delineation, five cells or polygons have been generated with two cells within the Bama location. Of these two cells, k_{31} houses demand point φ_1 and it is more effective to service it due to its closeness. Cell k_{32} contains three demand points φ_2, φ_3 and φ_4 which are quite some distance from it. From table 3.8, The distance of FOB (k_{31}) to the four demand points are
$H_{311} = 0.5cm(12.5km)$,
H_{312}=0.5cm (12.5km), H_{313}=2.1cm (52.5km) and H_{314}=1.9cm(47.5km),
From the distance evaluation, φ_1 and φ_2 are better served from k_{31} while φ_3 and φ_4 are better served from k_{32}. Although k_{31} and k_{32} both have advantages of distance over each other with respect to servicing φ_1, φ_2, φ_3 and φ_4, the engagement cost value shows that it is better to enlist the services of k_{32} with a lesser engagement cost rating of $411.43C_c$ over that of k_{31} of $921.5C_c$

5. Findings

1. Misallocation in Conventional Placement: In the conventional military FOB placement FOBs k33, k34, and k35 originally meant for the Bama domain are wrongly located and instead serve other service domains. Hence incorporating the Voronoi based heuristic algorithm significantly optimizes the allocation of Forward Operating Bases (FOBs), reducing unnecessary clustering and irregular placements

2. The placement of FOBs k33, k34, and k35 service Bama as it is currently results in unnecessarily long distances between them and their intended demand points. Also, Conventional FOBs k31 and k32 service ϕ3 and ϕ4 at much longer distances 62.5km and 65km (k31), and 52.5km and 47.5km (k32). Integrating the developed model would help to eliminate logistical bottlenecks caused by poor FOB placement and reduces the overall time and distance for resupply missions

3. The current conventional placement of FOBs k31 and k32, within Bama, are too close to each other, limiting their coverage and failing to reach distant demand points (ϕ3 and ϕ4) efficiently. In the proposed model, a single optimally located FOB (k3) effectively services demand points ϕ3 and ϕ4 at significantly shorter distances 30km and 22.5km respectively. Consequently, the Strategic placement of fewer, well-positioned FOBs improves the logistics replenishment network, leading to faster and more reliable supply lines to combat troops resulting in improved logistics efficiency.

4. It is more cost-effective for FOB k3 (from the proposed model) to service all Bama demand points, including those closer to conventional FOBs. The proposed FOB (k3) offers considerable cost savings 187.6Cc over k32 and 604.1Cc over k31 despite the closer proximity of k31 and k32 to other demand points (ϕ1 and ϕ2) when compared with the conventional placement.

5. Improved FOB location occasioned by the proposed model directly contributes to operational effectiveness in asymmetric warfare by ensuring continuous support to dispersed and mobile combat units. The algorithm ensures a more cost-efficient logistics system by minimizing redundant or overlapping FOB coverage areas.

6. The proposed model incorporates the characteristics of adaptability and scalability as it can be adapted over time as operational demand points shift, supporting dynamic adjustments is the scale of logistics planning.

7. Each FOB acts as a central logistics node within its Voronoi cell, serving surrounding demand points with minimal delay and maximum efficiency.

6. Conclusion

This study began by establishing the theoretical foundations for locating Forward Operating Bases (FOBs) within the context of asymmetric warfare. Drawing from military logistics literature, operational realities in North Eastern Nigeria, and the dynamics of non-linear conflicts, the study articulated the unique logistical challenges posed by asymmetric engagements. It highlighted the criticality of proximity, dispersion, and rapid resupply in sustaining operational tempo. The theoretical exposition established that FOB placement is not only a logistical necessity but also a strategic imperative influenced by factors such as terrain, threat variability, and demand point volatility. In pursuit of initial siting strategy, the research successfully applied a Voronoi-Based Heuristic Algorithm (VHA) to identify preliminary FOB locations. This heuristic method allowed for a scientifically informed setup of base positions by partitioning the operational area into Voronoi cells anchored on demand points. This approach addressed the issue of clustering and irregular placement that had previously hindered efficient resupply operations. The heuristic setup served as a necessary prelude to optimization, enabling the modeling process to begin from a relatively efficient base configuration that reflected real-world terrain and operational constraints. To optimize FOB locations, the study developed a novel model integrating resupply time, cost, and casualty risk into a composite objective function. This optimization model factored in variables such as Euclidean distances, casualty estimates, and threat exposure across multiple demand nodes. The result was a model capable of generating FOB placements that enhanced the overall efficiency and responsiveness of military logistics. The model demonstrated a measurable reduction in travel distances, supply costs, and risk, thereby contributing to a more robust and adaptive logistics network within the combat theatre. Lastly, the developed model was rigorously validated using empirical data from selected locations such as Bama in Borno State. Simulations and comparative analyses with existing military configurations confirmed the model's superiority in optimizing FOB placements. The optimized layout outperformed the conventional placement strategy in minimizing engagement cost and improving operational accessibility. The comparative analyses of Bama revealed that the optimized model significantly reduced average resupply distances from 30 km to 22.5 km (a 25% improvement) and the number of ambushes along logistics routes in one test area (Bama) dropped from 6 to 2 per 50 trips (a 66.7% reduction). Additionally, the engagement casualty cost estimator decreased from N412.2Cc to N317.9Cc, reflecting a 22.9% reduction in combat-related losses. The model also demonstrated that only 8 well-positioned FOBs were needed compared to 13 under the existing These findings affirm the model's potential application for real-time decision-making in logistics planning and underscore its value as a strategic tool for the Nigerian Army in asymmetric warfare scenarios.

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